Multipulse PPM on Memoryless Channels[1]

Jon Hamkins, Bruce Moision
Jet Propulsion Laboratory
California Institute of Technology
email: {jon.hamkins, bruce.moision}@jpl.nasa.gov

Abstract — **We examine several properties of n-pulse pulse position modulation (PPM) [1] on memoryless channels. We derive the maximum likelihood decision rule and an exact expression for the symbol error rate for $n \geq 1$, generalizing previous results [2, 3]. A capacity comparison indicates that multipulse PPM does not produce appreciable gains over conventional PPM except at high average power.**

I. Maximum Likelihood Detection of MPPM

In pulse position modulation (PPM), information is encoded by mapping each group of $\log_2(M)$ bits to a binary M-vector (M slots) with Hamming weight 1. Multipulse PPM [1] is a generalization of PPM in which each M-vector has Hamming weight n, $n \geq 1$.

Let $\{\mathbf{x}_1, \ldots, \mathbf{x}_{\binom{M}{n}}\}$ be the set of n-pulse M-slot PPM symbols. Each $\mathbf{x}_i = (x_{i,1}, \ldots, x_{i,M})$ is an M-vector with n '1's and $M-n$ '0's. Let $\mathcal{I}_k$ denote the set of indices for which $\mathbf{x}_k$ has a '1'. Let $\mathbf{Y} = (Y_1, \ldots, Y_M)$ denote the slot values received from the channel. Assuming the underlying channel is memoryless, we let $p_1(y)$ and $p_0(y)$ denote the conditional probability that a received slot has value y given a '1' (pulse) or '0' (no pulse), respectively, is transmitted in the slot. Let $P_1(y)$ ($P_0(y)$) denote the cumulative distributions, i.e., the probability that a received signal (nonsignal) slot has value less than or equal to y. We denote the log-likelihood ratio by $\Lambda(y) = \log\left[\frac{p_1(y)}{p_0(y)}\right]$.

Theorem 1. *If $\Lambda(y)$ is monotonically increasing in y, then the maximum likelihood (ML) rule for detection of n-pulse PPM given $\mathbf{Y} = \mathbf{y}$ is $\hat{\mathbf{X}} = \max_k \sum_{i \in \mathcal{I}_k} y_i$.*

Proof. The likelihood of receiving $\mathbf{Y} = \mathbf{y}$, given $\mathbf{X} = \mathbf{x}_k$, is

$$P_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x}_k) = \left(\prod_{i \in \mathcal{I}_k} \frac{p_1(y_i)}{p_0(y_i)}\right)\left(\prod_{j=1}^{M} p_0(y_j)\right)$$

Thus, $\hat{\mathbf{X}} = \max_k \log[P_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x}_k)] = \max_k \sum_{i \in \mathcal{I}_k} \Lambda(y_i)$, and the result follows by the monotonicity of $\Lambda(y)$. □

Theorem 1 generalizes a result for the Poission channel [4] to any channel with a monotonic log-likelihood ratio, including several channels of practical interest (e.g., Gaussian and Webb-McIntyre-Conradi).

II. The Symbol Error Rate

Theorem 2. *The symbol error rate (SER) of ML-detected n-pulse M-PPM on a discrete memoryless channel is $1 - \sum_y \sum_{l=0}^{M-n} \sum_{m=1}^{n} I(l,m)\alpha(l,y)\beta(m,y)$ where $\alpha(l,y) = \binom{M-n}{l} p_0(y)^l (P_0(y) - p_0(y))^{M-n-l}$ and $\beta(m,y) = \binom{n}{m} p_1(y)^m (1 - P_1(y))^{n-m}$.*

[1]The work described was funded by the Interplanetary Network Directorate Technology Program and performed at the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration.

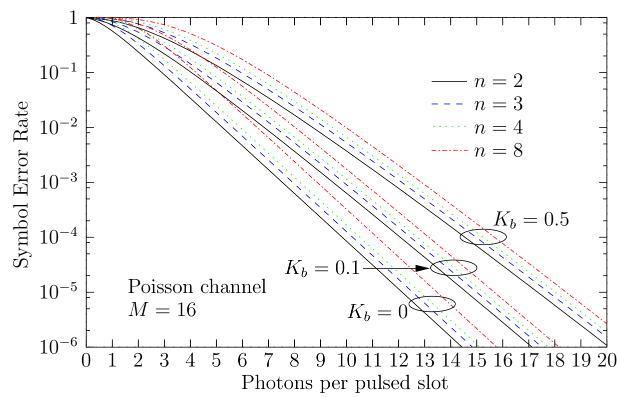

Figure 1: The performance of n-pulse 16-slot PPM.

This generalizes the $n \leq 2$ results of [2, 3], and the formula evaluation is more numerically stable as well. The SER of ML-detected n-pulse 16-slot PPM on a Poisson channel with mean $K_s + K_b$ (K_b) per signal (nonsignal) slot, respectively, is shown in Fig. 1. For each plot-point, a linux PC took about 0.03 seconds to compute all summation terms distinguishable from zero.

III. Capacity of Constrained Channels

Theorem 3. *For a given bandwidth, average power, and peak power, the supremum over n and M of the capacity of n-pulse M-slot PPM equals the capacity of on-off-keying (OOK) constrained to an equivalent duty-cycle.*

For low average signal power, the capacity of PPM is nearly equal to that of duty-cycle constrained OOK [5], and thus multipulse PPM offers no advantage over conventional PPM in this regime. Multipulse PPM begins to offer significant improvements— up to a doubling of capacity— over conventional M-slot PPM when the optimal M satisfies $M \leq 8$, which occurs when the average power is high.

References

[1] H. Sugiyama and K. Nosu, "MPPM: A method for improving the band-utilization efficency in optical PPM," *Journal of Lightwave Technology*, vol. 7, pp. 465–471, Mar. 1989.

[2] R. M. Gagliardi and S. Karp, *Optical Communication.* New York, NY: John Wiley and Sons, Inc., 1976.

[3] M. Simon and V. Vilnrotter, "Multi-pulse PPM signaling for optical communication with direct detection," *IPN Progress Report*, vol. 42–155, pp. 1–22, Nov. 2003.

[4] C. N. Georghiades, "Modulation and coding for throughput-efficient optical systems," *IEEE Trans. Inform. Theory*, vol. 40, pp. 1313–1326, Sept. 1994.

[5] B. Moision and J. Hamkins, "Deep-space optical communications downlink budget: Modulation and coding," in *IPN Progress Report*, vol. 42-154, Aug. 2003.